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PROBLEMS AND SOLUTIONS.

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PROBLEMS FOR SOLUTION.

[N.B. The editorial work of this department would be greatly facilitated if, on sending in problems, the proposers would also enclose their solutions—*when they have them.* If a problem proposed is not original the proposer is requested *invariably* to state the fact and to give an exact reference to the source.]

2876. Proposed by H. S. UHLER, Yale University.

Let a, d, n, and r denote respectively the refracting angle of a prism, the total deviation of the ray (in a principal plane) produced by the prism, the relative index of refraction of the material of the prism with respect to the single surrounding medium, and the angle of refraction at the first or incidence face of the prism.

(*a*) Deduce the (new) formula given below. (*b*) Use this formula to show (without the calculus) that the deviation has the least value when the ray is symmetrically situated with respect to the two refracting faces. (*c*) Show that, for minimum deviation, the formula reduces to the classical laboratory expression for determining the index of refraction.

$$\frac{\sin^2 \frac{1}{2}(a+d)}{\sin^2 \frac{1}{2}a} = n^2 + \frac{\sin \frac{1}{2}d \sin (a+\frac{1}{2}d) \sin^2 \frac{1}{2}(a+d) \sin^2 (r-\frac{1}{2}a)}{[\sin^2 \frac{1}{2}a \cos^2 \frac{1}{2}(a+d) \cos^2 (r-\frac{1}{2}a)+\cos^2 \frac{1}{2}a \sin^2 \frac{1}{2}(a+d) \sin^2 (r-\frac{1}{2}a)] \sin^2 \frac{1}{2}a}.$$

2877. Proposed by J. B. REYNOLDS, Lehigh University.

A particle slides down the rough arc of a cardioid, $r = a(1 - \cos \theta)$, which lies in a vertical plane, the initial line being horizontal. If the coefficient of friction, μ, equals $\frac{1}{3}$, find θ for the point where the particle leaves the curve, if it starts at $\theta = 90°$.

2878. Proposed by R. S. HOAR, Fort Banks, Mass.

Consider the integers $0, 1, 2, 3 \cdots n-1, n$. Consider all possible permutations of combinations of these taken r at a time, allowing any integer to occur more than once. Select from these permutations all groups the sum of whose integers is n. Form the reciprocal of the product of the factorials of the r integers of each of these selected groups. Then the sum of all of these reciprocals will equal $r^n/n!$. Prove that this must be so.

Example. $n = 2$, $r = 3$.

$$\frac{1}{2!0!0!} + \frac{1}{0!0!2!} + \frac{1}{0!2!0!} + \frac{1}{1!0!1!} + \frac{1}{1!1!0!} + \frac{1}{0!1!1!} = \frac{3^2}{2!}, \text{ if } 0! = 1.$$

2879. Proposed by E. J. OGLESBY, Washington Square College.

Given the values of $U_{5:9}$, $U_{5:10}$, $U_{5:11}$, $U_{6:9}$, $U_{6:10}$, $U_{6:11}$, $U_{7:9}$, $U_{7:10}$, $U_{7:11}$ where $U_{h:k} = \sqrt{hk}$, find the value of $U_{6.2:9.3}$ by interpolation.

2880. Proposed by SIDNEY DORB, Detroit, Mich.

Solve the simultaneous equations: $xy = 2$,

$$\left(3 - \frac{6y}{x-y}\right)^2 + \left(3 - \frac{6y}{x+y}\right)^2 = 82.$$

2881. Proposed by E. B. ESCOTT, Oak Park, Ill.

If, in the polynomial $X^3 - 2$, we substitute $x^2 + x - 4$ for X, the given expression can be factored, that is, $X^3 - 2 \equiv (x^3 + 3x^2 - 3x - 11)(x^3 - 6x + 6)$. Find a substitution for X so that the polynomial $X^3 + pX^2 + qX + r$ may be factored.

2882. Proposed by E. T. BELL, University of Washington.

Solve each of the following systems of finite difference equations, giving x_n, y_n, z_n as explicit functions of x_0, y_0, z_0, n:

$$x_n = x_{n-1}(y_{n-1}^3 - z_{n-1}^3), \qquad y_n = y_{n-1}(z_{n-1}^3 - x_{n-1}^3), \qquad z_n = z_{n-1}(x_{n-1}^3 - y_{n-1}^3); \tag{1}$$

$$\left\{\begin{aligned} x_n &= x_{n-1}^6 y_{n-1}^3 + y_{n-1}^6 z_{n-1}^3 + z_{n-1}^6 x_{n-1}^3 - 3x_{n-1}^3 y_{n-1}^3 z_{n-1}^3, \\ y_n &= x_{n-1}^3 y_{n-1}^6 + y_{n-1}^3 z_{n-1}^6 + z_{n-1}^3 x_{n-1}^6 - 3x_{n-1}^3 y_{n-1}^3 z_{n-1}^3, \\ z_n &= x_{n-1} y_{n-1} z_{n-1}(x_{n-1}^6 y_{n-1}^6 + z_{n-1}^6 - y_{n-1}^3 z_{n-1}^3 - z_{n-1}^3 x_{n-1}^3 - x_{n-1}^3 y_{n-1}^3). \end{aligned}\right. \tag{2}$$

A solution of either would be acceptable. From the theory of plane cubic curves a geometrical construction for the solution may be given. Also both systems have solutions in terms of elliptic functions. Do solutions exist, and how are they found, without using elliptic functions?

PROBLEMS—NOTES.

4. On page 83 of *Revista de Matematicas y Fisicas Elementales*, published at Buenos Aires, August, 1920, Problem 2785 of this MONTHLY [*1919*, 366], and the solution [*1920*, 237] of its proposer, Professor W. H. ECHOLS, are translated into Spanish. The problem in question is: "If on the sides, as bases, of any closed plane polygon, there be constructed similar triangles similarly placed, all outward or all inward, then the centroid of the vertices of these triangles coincides with the centroid of the corners of the polygon."

5. In *The Ladies Diary: or the Women's Almanack*, for 1739, the following problem (207) was proposed: "There came three Dutchmen of my acquaintance to see me, being lately married; they brought their wives with them. The men's names were Hendrick, Claas, and Cornelius; the women's Geertruii, Catriin, and Anna; but I forgot the name of each man's wife. They told me they had been at market to buy hogs; each person bought as many hogs as they gave shillings for each hog: Hendrick bought 23 hogs more than Catriin, and Claas bought 11 more than Geertruii; likewise, each man laid out 3 guineas more than his wife. I desire to know the name of each man's wife?" Solutions were published in the *Diary* for 1740[1]; in one of them it is remarked that "the number of hogs, the three men and their respective wives bought, will be express'd by three pair of numb. the difference of whose squares must be 63."

6. Dr. OTTO KLOTZ, of the Dominion Observatory, Ottawa, suggested the following solution of the problem, to construct with ruler and compasses lines equal in length to the reciprocals of the positive integers: On AB, the unit of length, construct the square $ABCD$ (the reader is requested to draw the figure). Let the diagonals BD, AC intersect in E. Through E draw $EF \parallel DA$ and meeting AB in F; join CF meeting DB in G. Through G draw $GH \parallel DA$ and meeting AB in H; join CH meeting DB in I. Through I draw $IK \parallel DA$ and meeting AB in K; and so on. $BF = 1/2$; $BH = 1/3$; $BK = 1/4$; etc.

Dr. Klotz's construction would be equally valid if a parallelogram replaced the square $ABCD$. Indeed both of the constructions may be looked upon as

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[1] These solutions were reprinted in *The Mathematical Questions proposed in the Ladies' Diary* . . . by Thomas Leybourn, London, 1817, volume 1, pp. 280–282; *The Diarian Miscellany* . . . by Cha. Hutton, London, 1775, volume 2, pp. 104–106. One of the solutions was reprinted in *The Diarian Repository; or Mathematical Register* . . . by a Society of Mathematicians, London, 1774, pp. 359–360.